DYNAMICS OF ICE–ROCK BARRIERS UNDER CONDITIONS OF FREEZING OF FILTERING ROCKS

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Nonstationary heat transport under conditions of freezing of filtering soils is studied using a mathematical model which takes into account an arbitrary distribution of sources of cold.

The study of the dynamics of growth of an ice–rock body around a system of freezing boreholes is an important problem in modern hydraulic engineering [1, 2].

This work is concerned with the construction of an approximate method for studying the growth dynamics of an ice–rock body under the conditions of filtration flow.

Analogous problems were studied in [3, 4]. A significant drawback of those studies is that in the solution of the corresponding problems conductive heat transport along the streamlines was neglected. In addition, in [4] the ice–rock body is represented in a form which approximates a real body with a quite large error.

We shall study the plane problem, which is equivalent to the assumption that the depth of the borehole is infinite and the temperature field, the filtration velocity field, and the pressure field are uniform along the borehole.

We introduce the coordinate system (x, y) in the plane perpendicular to the axes of the borehole. We use the following notation: (x_k, y_k) denotes the coordinates of the centers of the boreholes; R_b, radius of the borehole; V_∞, t_∞, velocity and temperature of the filtration flow at infinity; t_c, temperature of the cooling agent; t_p, temperature of the phase transition; and τ_0, time at which the ice–rock barriers first join. We shall confine our attention to the study of the process in the time interval from 0 to τ_0. In view of the fact that the filtration motion is stabilized much more rapidly than the temperature field, the basic system of equations can be written in the form

$$\begin{gathered}\operatorname{div} V=0,\quad V=-\varkappa \operatorname{grad} P,\quad (x,\ y)\in R^2/\bigcup_k \overline{D}_k,\\ \frac{\partial t_t}{\partial \tau}=a_t \operatorname{div}(\operatorname{grad} t_t)-V\operatorname{grad} t_t,\ (x,\ y)\in R^2/\bigcup_k \overline{D}_k,\\ \frac{\partial t_k}{\partial \tau}=a_f \operatorname{div}(\operatorname{grad} t_k),\quad (x,\ y)\in D_k,\ \ \lambda_f \frac{\partial t_k}{\partial n}\Big|_{\partial D_k}-\lambda_t \frac{\partial t_t}{\partial n}\Big|_{\partial D_k}=L\frac{dn}{d\tau},\\ t_k|_{\Gamma_k}=t_b,\quad t_k|_{\partial D_k}=t_t|_{\partial D_k}=t_p,\quad V=V_\infty,\quad t_t=t_\infty,\\ x^2+y^2\to\infty,\quad t_t=t_\infty,\quad \partial D_k=\Gamma_k,\quad \tau=0,\end{gathered} \tag{1}$$

where D_k is the region occupied by the frozen soil around the k-th borehole; ∂D_k is the boundary of the region D_k; and $\overline{D}_k = D_k \cup \partial D_k$.

Choosing for the characteristic scales the quantities

$$l=a_f K_p/V_\infty,\quad t_b-t_p,\quad t_\infty-t_p,\ V_\infty,$$
$$p=rV_\infty/\varkappa,\quad r=a_t/K_b V_\infty,\quad K_b=\frac{c_w\rho_w}{c_t\rho_t}$$

and introducing the dimensionless variables $\tau' = \tau a_f/r^2$, $V' = V/V_\infty$,

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$$x'=\frac{x}{r},\quad y'=\frac{y}{r},\quad (x,\ y)\in R^2/\bigcup_k \overline{D}_k,\quad P'=P/p,$$

$$x'=\frac{x}{l},\quad y'=\frac{y}{l},\quad (x,\ y)\in D_k,\quad K_a=\frac{a_t}{a_f},\tag{2}$$

$$\Theta_k=\frac{t_k-t_p}{t_b-t_p},\quad \Theta=\frac{t_t-t_p}{t_\infty-t_p};\ K_p=L/c_t\rho_t(t_\infty-t_p),\ \mathrm{Ko}=L/c_f\rho_f\,(t_b-t_p).$$

In what follows, we omit the prime on the dimensionless variables. The system (1) is written in the form

$$\operatorname{div} V=0,\tag{3}$$

$$V=-\operatorname{grad} P,\tag{4}$$

$$\frac{K_a}{K_p^2}\frac{\partial\Theta}{\partial\tau}+V\operatorname{grad}\Theta=\operatorname{div}(\operatorname{grad}\Theta),\tag{5}$$

$$\frac{\partial\Theta_k}{\partial\tau}=\operatorname{div}(\operatorname{grad}\Theta_k),\tag{6}$$

$$\begin{gathered}\frac{1}{\mathrm{Ko}}\frac{\partial\Theta_k}{\partial n}\Big|_{\partial D_k}-\frac{\partial\Theta}{\partial n}\Big|_{\partial D_k}=\frac{dn}{d\tau},\\ \Theta_k|_{\Gamma_k}=1,\quad \Theta|_{\partial D_k}=\Theta_k|_{\partial D_k}=0,\\ V=1,\quad \Theta=1\quad x^2+y^2\to\infty,\\ \Theta=1\quad \partial D_k=\Gamma_k,\quad \tau=0.\end{gathered}\tag{7}$$

The parameter K_p is much greater than unity for moist rock. Therefore, the first term on the left side of Eq. (5) can be dropped. In addition, based on the experimental data of [1] it may be assumed that at each moment in time the form of the ice–rock body around a separate borehole is a circle of radius R_k, shifted relative to the axis of the borehole by an amount e_k. The slope angle between the velocity vector V_∞ and the straight line passing through the center of the circles is denoted by α_k.

Introducing in a standard manner the complex potential of the flow and applying to Eq. (5) the Boussinesq transformation, we obtain

$$\begin{gathered}\frac{\partial\Theta}{\partial\varphi}=\frac{\partial^2\Theta}{\partial\varphi^2}+\frac{\partial^2\Theta}{\partial\psi^2}\ (\varphi,\ \psi)\in R^2/\bigcup_k L_k,\quad \Theta=0,\ (\varphi,\ \psi)\in L_k,\\ \Theta=1,\quad \varphi^2+\psi^2\to\infty,\quad L_k=\{\psi=\psi_k,\ \gamma_k\leqslant\varphi\leqslant\nu_k\}.\end{gathered}\tag{8}$$

The planes W, with the exception of the segments L_k, correspond to the physical plane Z from which the circles $\overline{D}_k$ are cut out. Thus to solve Eq. (5) it is necessary to know the streaming potential of the given array of circular profiles. We shall call this problem "problem 1." The solution of the problem (5) is also of interest in itself, so that we shall call it "problem 2." Possible approaches to the solution of this problem are analyzed in [3, 5].

We shall consider the solution of the problem (6) and (7). Since it is nonlinear, it is difficult to solve it in a closed form even in the one-dimensional case, so that we shall employ the integral method [3, 6-8].

We introduce the system of coordinates $(\xi,\ \eta)$, fixed to the k-th wells as shown in Fig. 1. We write $f_{ij}=\xi^i\eta^j$. Multiplying (6) by f_{ij} and integrating over the region D_k we obtain

$$\frac{d}{d\tau}\left\{\int\limits_{D_k}\!\!\int f_{ij}(\Theta_k-\mathrm{Ko})\,d\Omega\right\}=\int\limits_{D_k}\Theta_k\Delta f_{ij}d\Omega+\int\limits_{\Gamma_k}\left(\frac{\partial f_{ij}}{\partial n}-f_{ij}\frac{\partial\Theta_k}{\partial n}\right)ds+\mathrm{Ko}\int\limits_{\partial D_k}f_{ij}\frac{\partial\Theta}{\partial n}\,ds,\tag{9}$$

where n is the outer normal. Choosing the temperature profile in the form [9]

$$\Theta_k=\frac{1}{2C_1}\{\ln(\xi^2+\eta^2+2(h+b)C_4+(h+b)^2-\ln(\xi^2+\eta^2+2(h-b)C_4+(h-b)^2)+2C_2\}-C_3,$$

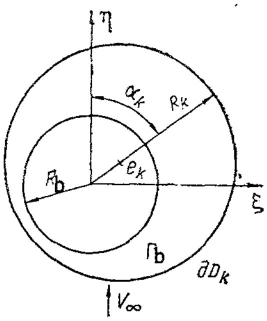

Fig. 1. Ice–rock body formed around a borehole.

$$C_1 = \frac{t_c - t_\Phi}{t_c} \ln \frac{R_k (h + b)}{R_c (H + b)}, \quad C_2 = -\ln \frac{H + b}{R_k}, \quad C_3 = \frac{t_\Phi}{t_c - t_\Phi},$$

$$C_4 = \eta \cos \alpha_k + \xi \sin \alpha_k, \quad h = \frac{R_k^2 - R_c^2 - e_k^2}{2e_k},$$

$$H = \frac{R_k^2 - R_c^2 + e_k^2}{2e_k}, \quad b^2 = H^2 - R_k^2 = h^2 - R_c^2$$

and using the three integral relations from the system (9) with i = 0 j = 0, i = 0 j = 1, and i = 1 j = 0, we obtain a system of ordinary differential equations for determining the unknown functions R_k, e_k, α_k:

$$\frac{d}{d\tau} \int_{D_k} (\Theta_k - \mathrm{Ko})\, d\Omega = -\int_{\Gamma_k} \frac{\partial \Theta_k}{\partial n} ds + \mathrm{Ko}\, Q_1,$$

$$\frac{d}{d\tau} \int_{D_k} \xi (\Theta_k - \mathrm{Ko})\, d\Omega = \int_{\Gamma_k} \left(\frac{\partial \xi}{\partial \eta} - \xi \frac{\partial \Theta_k}{\partial n} \right) ds + \mathrm{Ko}\, Q_2,$$

$$\frac{d}{d\tau} \int_{D_k} \eta (\Theta_k - \mathrm{Ko})\, d\Omega = \int_{\Gamma_k} \left(\frac{\partial \eta}{\partial n} - \eta \frac{\partial \Theta_k}{\partial n} \right) ds + \mathrm{Ko}\, Q_3, \tag{10}$$

$$Q_1 = \int_{\partial D_k} \frac{\partial \Theta}{\partial n} ds, \quad Q_2 = \int_{\partial D_k} \xi \frac{\partial \Theta}{\partial n} ds,$$

$$Q_3 = \int_{\partial D_k} \eta \frac{\partial \Theta}{\partial n} ds.$$

Thus, the problem of studying the growth dynamics of an ice–rock body around a system of columns subject to freezing up has actually been reduced to the construction of the complex potential of a flow and finding the temperature field in the W plane. Problem 1 has been studied by many authors, so that we shall not consider it. An extensive bibliography is presented in [10].

To solve the problem 2 we shall examine the Fourier transformation of Eq. (8) with respect to the variable ψ. We make the substitution T = Θ − 1. Omitting the calculations, we write out the system of integral equations for determining the unknown heat fluxes:

$$2\pi = \sum_{k=1}^{N} \int_{\gamma_k}^{\nu_k} \exp\left(\frac{\varphi - \sigma}{2} \right) \mu_k(\sigma) K_0 \left(\frac{1}{2} \sqrt{(\varphi - \sigma)^2 + (\psi_k - \psi_j)^2} \right) d\sigma,$$

$$j = \overline{1, N}; \quad \varphi \in [\gamma_j, \nu_j],$$

where

$$\mu_k = \left. \frac{\partial \Theta}{\partial \psi} \right|_{\psi = \psi_k + 0} - \left. \frac{\partial \Theta}{\partial \psi} \right|_{\psi = \psi_k - 0}; \tag{11}$$

and $K_0(z)$ is the MacDonald function.

To solve the system of integral equations (11) it is necessary to know the behavior of the functions μ_k at the points γ_k, ν_k, corresponding to the critical points for the fluid flow.

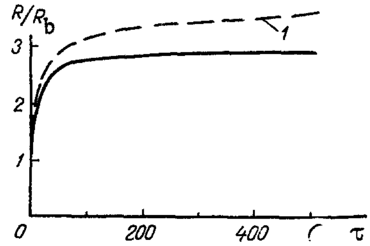

Fig. 2. Dependence of the average radius of the ice-rock barrier on time for one borehole and for a gallery of boreholes, arranged perpendicularly to the incident flow. The solid line corresponds to a single column: 1) $d = 10R_c$.

We study a neighborhood of the critical point in the plane of the potential and in the physical plane. Following a circuit around the point Z_k in the physical plane the vector $Z - Z_k$ turns by an angle π, while the vector $W - W_k$ turns by 2π. It can be shown that the conformal nature of the mapping at the point W_k breaks down and $dW/dZ = O(\sqrt{W - W_k})$. Writing the heat flux in the form

$$\left|\frac{\partial\Theta}{\partial n}\right| = \left|\frac{\partial\Theta}{\partial\psi}\right|\left|\frac{dW}{dZ}\right|$$

and taking into account the fact that at the critical points it is finite, we obtain

$$\mu_k = O\left(\frac{1}{\sqrt{W - W_k}}\right). \tag{12}$$

Thus the calculation of the parameters of the ice-rock barriers must be performed using the following scheme:

1) determine W;

2) solve the system of integral equations, and find

$$\partial\Theta/\partial\psi|_{\psi=\psi_k\pm 0};$$

3) from the solution of the system of ordinary differential equations find the parameters of the ice-rock bodies.

To illustrate the scheme presented above we shall study the problem of freezing of the soil for one column. As is well known [2], this solution is also used to calculate the outer contour of the ring-shaped barrier used for the passage of well-shafts.

The complex flow potential in this case is easy to write down; it is the Zhukovskii potential. The integral equation obtained from the system (11) can be solved by the Wiener-Hopf method, analogous to the manner in which this is done in [11]. Because of the limited space available in this paper we cannot present the detailed calculations, so we shall merely write out the result:

$$\left|\frac{\partial\Theta}{\partial\psi}\right| = \frac{1}{\sqrt{\pi(a+\varphi)}} + \frac{\exp(\varphi)K_0(a)}{2\pi\sqrt{\pi(a-\varphi)}} - \frac{2}{\pi}\exp(\varphi - a)\int_0^\infty \exp(-2ap^2)\,\mathrm{erfc}\sqrt{(p^2+1)(a-\varphi)}\,dp. \tag{13}$$

This result holds for all a satisfying the condition $\pi \leqq -\mathrm{Ei}(-a)$, where a is the half-width of the cut in the plane of the potential. We note that the first term in (13) is the solution of the problem (8) under the assumption that the thermal conductivity along the streamlines can be neglected. It is easy to see that this assumption leads to the fact that the heat flow at the point of convergence of the fluid flow approaches zero, which is unrealistic. It can be shown that further refinement of (13) only gives small corrections, and is insignificant for the estimate presented above.

Figures 2 and 3 show the results of calculations for the following values of the parameters: $R_e = 1.875\cdot 10^{-2}$ m, $V_\infty = 3\cdot 10^{-5}$ m/sec, $t_c = 253$°K, $t_\infty = 293$°K, $\lambda_t = 2.21$ W/m·°K, $\lambda_m = 2.43$ W/m·°K, $a_t = 0.8\cdot 10^{-5}$ m²/sec, $a_m = 1.3\cdot 10^{-5}$ m²/sec, $L = 16.3\cdot 10^7$ J/m³.

It is evident from the graph of the behavior of the functions R/R_b, e/R_b that in the limit $\tau \to \infty$ they approach their limiting values R_∞/R_c, e_∞/R_c. In addition, calculations were

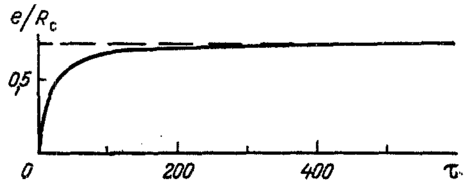

Fig. 3. Eccentricity as a function of time for one borehole.

also carried out for the case when the freezing is done by a gallery of boreholes. In this case, the integral equation corresponding to the system (11) was solved numerically. The results of the calculations show that if the distance between the columns is significant (>20 R_c), then the interaction is not important over real time intervals.

For operational evaluation of the parameters of freezing systems, providing for joining of ice–rock barriers, to a first approximation the result obtained for one borehole can be used. Setting in Eqs. (8) in the limit $\tau \to \infty$ the time derivatives equal to zero, we find

$$Q_2 + Q_1\left(R\exp\left(-\frac{2}{\Phi Q_1}\right) - 1\right)/e = 0,$$

$$Q_1 = \int_{-a}^{a}\left|\frac{\partial\Theta}{\partial\psi}\right| d\varphi, \quad Q_2 = \int_{-a}^{a}\frac{\varphi}{2}\left|\frac{\partial\Theta}{\partial\psi}\right| d\varphi + eQ_1, \tag{14}$$

$$e = (R\exp(2/\Phi Q_1) - 1)^{1/2}(R\exp(-2/\Phi Q_1) - 1)^{1/2},$$

$$\Phi = \frac{2\lambda_t t_\infty}{\lambda_f \pi t_c}\sqrt{\frac{\mathrm{Pe}}{\pi}}, \quad \mathrm{Pe} = \frac{R_b V_\infty}{a_t}\,\frac{c_w \rho_w}{c_t \rho_t},$$

$$R = R_\infty/R_b, \quad e = e_\infty/R_t,$$

where a = 2R.

For the joining criterion we can use

$$d \leqslant 2R_\infty, \tag{15}$$

i.e., the distance between the centers of the freezing boreholes must be less than or equal to the limiting diameter of the ice–rock barrier around a separate column. Thus based on the given value of d, according to (15), it is possible to select the required value of R_∞. Then, solving the transcendental equation, it is possible to calculate the value of the parameter Θ based on which it is possible to judge, for example, the temperature required in the borehole.

It is interesting to note that if in the formula (13) only the first term is retained, i.e., heat conduction along streamlines is neglected, then the formula for the eccentricity

$$e = \frac{5}{3}R - \sqrt{\frac{16}{9}R^2 + 1}$$

is practically identical to the analogous formula presented in [2].

NOTATION

a, coefficient of thermal diffusivity; λ, coefficient of thermal conductivity; ϰ, filtration coefficient; L, latent heat of fusion; c, specific heat capacity; ρ, density; r, characteristic size in the thawed zone; l, characteristic size in the frozen zone; V, velocity; P, pressure; p, characteristic pressure; t, temperature; τ, time; x and y, Cartesian coordinates; ξ, η, local coordinates in the frozen zone, defined in Fig. 1; Γ_k, boundary of the k-th borehole; ∂D_k, boundary of the ice–rock body; D_k, region occupied by the frozen soil around the k-th borehole; n, outer normal; Z = x + iy, a complex variable in the physical plane; W = φ + iψ, complex potential of the flow; Δ, Laplacian operator; γ_k, ν_k, ends of the cut in the W plane; ψ_k, value of the stream function at the k-th circle; τ_0, time at which the ice–rock barriers first join; R, radius of the circle; e, eccentricity; α, slope angle of the velocity vector at infinity relative to the straight line passing through the center of the circles. The dimensionless criteria, parameters, and functions are as follows: Θ_k, Θ, K_a, K_p, Ko, K_c are defined in (2); Q_1, Q_2, Q_3 are defined in (10) and (11); μ, unknown density in the integral equation; $K_0(Z)$, MacDonald's function; d, distance between the centers of the boreholes; ϕ, parameter from (14); Pe, Peclet's number from (14); Z_k and W_k, critical points of the flow in the corresponding planes. Indices: t, thawed zone; f, frozen zone; b, surface of the borehole; p, phase-transition surface; w, water; ∞, value at infinity; the prime in (2) denotes a dimensionless variable.

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